

## Worksheet 2

Prove the following statements

Write down what is the assumption/supposition and what are you trying to prove.

Remember definitions are your tools (or best friend)! Stick to the definitions.

1. Let  $a$ ,  $b$ , and  $c$  be integers. If  $a$  divides  $b$ , then  $a$  divides  $bc$ .

a. Assumption:

b. To prove:

c. Proof:

2. If  $x$  is an integer, then  $x^2 + x + 3$  is an odd integer.

a. Assumption:

b. To prove:

c. Proof:

3. The product of consecutive integers is an even integer.

a. Assumption:

b. To prove:

c. Proof:

4. Let  $x$  be an integer. If 4 does not divide  $x^2$ , then  $x$  is odd.

- a. Assumption:
- b. To prove:
- c. Proof:

5. Let  $x$  be an integer. If 8 does not divide  $x^2 - 1$ , then  $x$  is even.

- a. Assumption:
- b. To prove:
- c. Proof:

6. If  $a, b, c$  are integers such that  $a|b$  and  $a|c$ , then  $a|(b + c)$ .

- a. Assumption:
- b. To prove:
- c. Proof:

7. Let  $a, b$  and  $c$  be positive integers. The integer  $ac$  divides  $bc$  if and only if the integer  $a$  divides  $b$ .

a. Assumption:

b. To prove:

c. Proof:

8. Let  $x$  be a real number. The quadratic  $x^2 + 2x + 1 = 0$  if and only if  $x = -1$ .

a. Assumption:

b. To prove:

c. Proof:

9. Let  $x$  be an integer. If  $9x + 5$  is even, then  $x$  is odd.

a. Assumption:

b. To prove:

c. Proof:

10. Let  $x$  be an integer.  $x^2$  is even if and only if  $x$  is even.

- a. Assumption:
- b. To prove:
- c. Proof:

11. Let  $a, b$  be integers.  $ab$  is even if and only if  $a$  is even or  $b$  is even.

- a. Assumption:
- b. To prove:
- c. Proof:

12. Disprove by finding a counterexample:

For all real numbers  $a$  and  $b$ , if  $b^2 > a^2$ , then  $b > a$